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VISCOSITY OF STEAM AT HIGH TEMPERATURES AND PRESSURES

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Abstract—The viscosity of a real gas depends upon not only temperature but also pressure, namely it may be a kind of quantity of state, and connects with the other quantities of state by some relation. In order to make clear the dependence of viscosity of steam upon pressure and temperature from such a view point, this paper deals with the numerical treatment based on the experimental results by various investigators. And the new characteristic relation of steam between viscosity, the second virial coefficient and the compressibility factor was established.

NOMENCLATURE

B,	second virial coefficient, m^3/kg;
C,	third virial coefficient, m^6/kg^3;
$\bar{N}$,	number of molecules;
T,	absolute temperature, °K;
T_c,	critical temperature, °K;
T^*,	reduced temperature, T/T_c;
Y,	dimensionless quantity, $(-B/v)/(1 - z)$;
Z,	compressibility factor;
a, b, c,	constants;
b,	co-volume. $\frac{2}{3}\pi\bar{N}\sigma^3$;
v,	specific volume, m^3/kg;
y,	dimensionless quantity;
α, β,	constants;
γ,	specific weight, kg/m;
τ,	reciprocal absolute temperature, 1/°K;
σ,	collisional diameter;
η,	viscosity, kg s/m^2;
η_a,	viscosity at atmospheric pressure, kg s/m^2;
η_{ac},	viscosity at critical temperature and atmospheric pressure, kg s/m^2;
η^*,	reduced viscosity, η/η_a;
η_a^*,	reduced viscosity at atmospheric pressure, η_a/η_{ac}.

1. INTRODUCTION

THE VISCOSITY of steam is of fundamental importance in the fields of heat transfer and fluid dynamics. And although there were many experimental investigations on the viscosity of steam at atmospheric pressure, and also on the dependence of the viscosity of steam upon pressure hitherto, there were considerable discrepancies between their results by their experimental and calibrating methods. Recently, the considerably reliable data have been reported, for instance on the viscosity at atmospheric pressure by Shifrin [1] and Kestin *et al.* [2] and also on the viscosity at high pressures by Timrot [3], Whitelaw [4] and Mayinger [5].

Therefore, it may be obviously important to establish the relation between the viscosity of steam, temperature and pressure with consideration of the mutual differences of these results.

According to the simple kinetic theory of gas molecules, the viscosity of an ideal gas is the function of temperature only, but the viscosity of a real gas must be expressed as the function of temperature and pressure, namely it may be considered to be also a kind of quantities of state. The general correlation, however, between viscosity and another quantities of state has not yet been obtained theoretically.

In this paper, in order to make clear the dependence of the viscosity of steam upon pressure and temperature, the numerical treatment based on the experimental results by various investigators is carried out.

2. VISCOSITY AT ATMOSPHERIC PRESSURE

In order to determine the relation between the viscosity of steam at atmospheric pressure and temperature, the relation between the viscosity of steam η_a (kg s/m²) by the various authors at atmospheric pressure and the reduced temperature T^* ($T^* = T/T_c$; $T_c = 647{\cdot}3$°K, critical temperature) are shown in Fig. 1. Though there are some deviations and also some anomalous trends between these results, every result approximately satisfies the next linear relation in this figure

$$\log \eta_a = \alpha \log T^* + \log \beta. \qquad (1)$$

In this equation, β is equal to the viscosity of steam η_{ac} at the critical temperature T_c, and atmospheric pressure.

By substituting $\eta_a^* = \eta_a/\eta_{ac}$, the following correlation is obtained

$$\eta_a^* = T^{*\alpha}. \qquad (2)$$

From this figure, η_{ac} and α are determined as follows,

$$\eta_{ac} = 2{\cdot}325 \times 10^{-6}\ \text{kg s/m}^2, \quad \alpha = 1{\cdot}11.$$

Therefore,

$$\eta_a^* = T^{*1{\cdot}11} \qquad (3)$$

or

$$\eta_a = 1{\cdot}751 \times 10^{-9}\, T^{1{\cdot}11}. \qquad (4)$$

Fig. 2 shows the comparison with the various results at atmospheric pressure, by using the deviation per cent from the calculated value.

According to the simple reduction by means of the kinetic theory of gas molecules, the viscosity of a dilute gas is proportional to the inverse of collisional cross section of gas molecules and the square root of product of molecular mass and absolute temperature. Therefore, for such a gas, its collisional diameter is equal to the molecular diameter or independent of temperature, its viscosity is proportional to the square root of absolute temperature. But in the case of a real gas, the collisional diameter may change with temperature, namely it has been considered that the collisional diameter decreases, since the molecular velocity increases in accordance with temperature rise. Then the viscosity

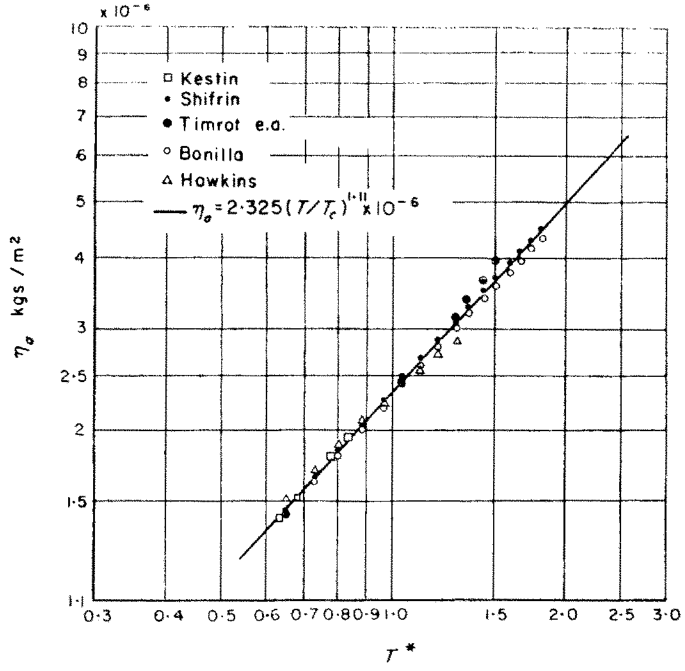

Fig. 1. Viscosity at atmospheric pressure.

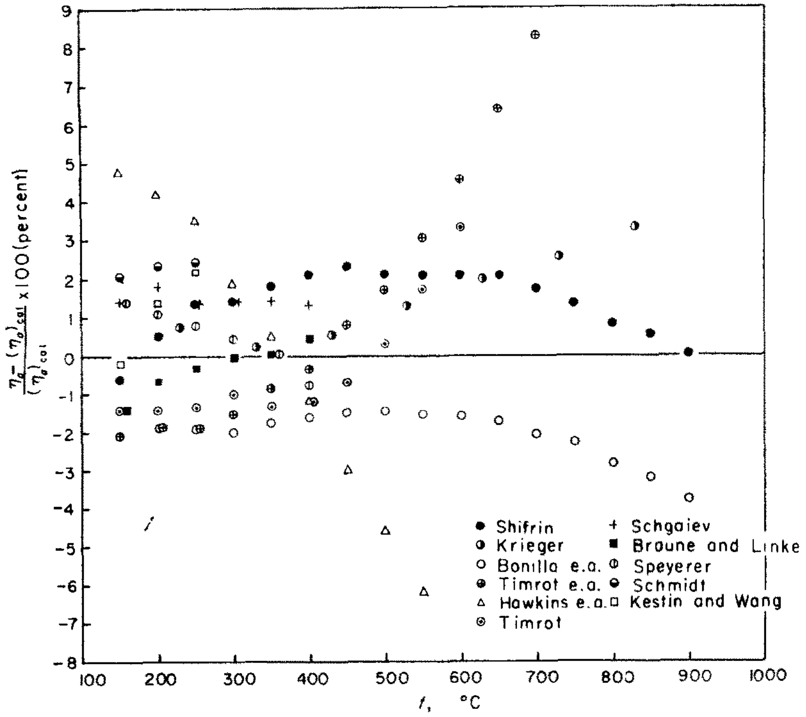

FIG. 2. Comparison of results at 1 atm.

of a real gas may not be exactly proportional to the square root of absolute temperature. For a dilute gas, especially a gas composed of nonpolar molecules, the viscosity and the other transport properties have been determined theoretically and the agreements with the experimental values are comparatively good, but such a treatment has not yet been succeeded for a gas composed of polar molecules.

However, also the visosity of a polar gas may be expressed as the function of the product of square root of absolute temperature and the inverse of collisional integral, for example, as Krieger [6] calculated by using the modified Stockmeyer potential equation for the interaction energy between a pair of polar molecules.

In the above mentioned equation (4), the viscosity of steam at atmospheric pressure is proportional to the 1·11-power of absolute temperature, then the collisional integral for steam may be approximately proportional to the −3/5-power of absolute temperature. But in the case that the viscosity is proportional to absolute temperature such as Shifrin's equation, the collisional integral may change proportional to the negative square root of absolute temperature.

However, in order to certify the behaviour of collisional integral for polar gases, the more accurate experimental data and the more rigorous kinetic theory are desired.

From the comparison as shown in Fig. 2, it follows the evident fact that the values due to Timrot *et al.* and those due to Hawkins *et al.* have different trends to each other, and the former deviates from the latter by from +7 per cent at 150°C to −9 per cent at 550°C, and the results by Bonilla *et al.* and Shifrin's results differ by a constant amount of −3·8 per cent.

As mentioned above, Krieger calculated the viscosity by using the modified Stockmayer potential function in the range 300–3000°K at atmospheric pressure. But the reliability of his results is in question, since the intermolecular constants were determined as a mean of four points from different investigators, namely

Smith, Schgaiev, Timrot and Keyes, and there are some questions in his treatment with regard to the collisional integral especially in the case of gases having comparatively large dipole moment.

The dimensionless formula (3) between the viscosity of steam and temperature agrees reasonably well with the many data published previously as shown in Fig. 2.

3. VISCOSITY AT HIGH PRESSURES

Although several measurements were carried out in order to make clear the dependence of the viscosity of steam upon pressure by various methods since Speyerer had attempted in 1925, for example by Schiller, Schgaiev, Sigwart, Hawkins, Jackson, Osborne, Timrot, Kjelland-Fosterud and recently Whitelaw and Mayinger and so on, the steam viscosity by these measurements can be separated into two groups on the effect of pressure and there exists considerable quantitative difference between them. It is considered that such discrepancy is due to the invalidity of the fluid dynamical correction, and especially the results by Speyerer, Schiller and Hawkins *et al.* are considered to be unreliable in high pressure range, in which the effect of pressure is too large, as it has been pointed out by Timrot in 1940.

Recently, the determination of the viscosity of steam at supercritical pressures and temperatures has been carried out by Whitelaw with a modified Rankine type viscometer, which was designed by Kjelland-Fosterud, and the standard deviation of his final results is stated to be ±1·5 per cent. And Mayinger has reported the experimental values of the viscosity of water and steam carried out by a capillary method at temperatures up to 700°C, and pressures up to 800 kg/cm^2 and maximum uncertainty in his results is estimated at ±1·5 per cent.

As mentioned above, we do not take into consideration here the values by Speyerer, Schiller and Hawkins *et al.* and only the results by Timrot, Sigwart, Whitelaw and Mayinger are limited to be used for the following consideration. Kestin measured the viscosity of steam along the four isotherms from about atmospheric pressure to saturation pressure and his results revealed an anomaly in which the pressure effect is negative, decreasing with increasing temperature down to zero at 270°C. This tendency of the viscosity of steam has been discovered by Moszynski. Although the values of these two experiments are in excellent agreement within the stated accuracy, this tendency must be confirmed by the more accurate experiment and the more rigorous theory. And in this paper, the authors do not consider the pressure effect on the viscosity of steam in the temperature region lower than 300°C.

Enskog developed a kinetic theory of a dense gas made up of rigid spherical molecules, and the results of the Enskog theory [7] on the viscosity of a dense gas is shown as follows,

$$\frac{\eta v}{\eta_o b_o} = \frac{1}{y} + 0{\cdot}8 + 0{\cdot}761 y \qquad (5)$$

where, η_o is the zero-pressure value of viscosity, v is the specific volume, b_o is the co-volume ($=\frac{2}{3}\pi \bar{N} \sigma^3$, σ is the collisional diameter, $\bar{N}$ is the number of molecules), and the parameter y is a dimensionless quantity. Namely, the reduced viscosity times the reduced volume is expressed as a function of the parameter y only.

Though this is a discussion with regard to the viscosity of a dense gas made up of rigid spherical molecules, it is considered that there is a similar relation between the viscosity and the quantity of state also for a gas made up of polar molecules.

In this report, the relation between the compressibility factor as a fundamental quantity of state and the experimental values of steam viscosity is treated as follows.

Fig. 3 shows the relation between $(\eta/\eta_a)v$ and $(1 - Z)$, where η_a is the viscosity of steam at atmospheric pressure, v and Z are the specific volume and the compressibility factor of steam respectively.

It is interesting that the various experimental values satisfy a linear relation in this figure under constant temperature, and then their inclinations are nearly equal to minus one.

Therefore, it may be considered that $(\eta/\eta_a)v$ is the function of $(1 - Z)$ and temperature, except for the extremely high pressures and low temperature range.

If the virial expansion form is used as an

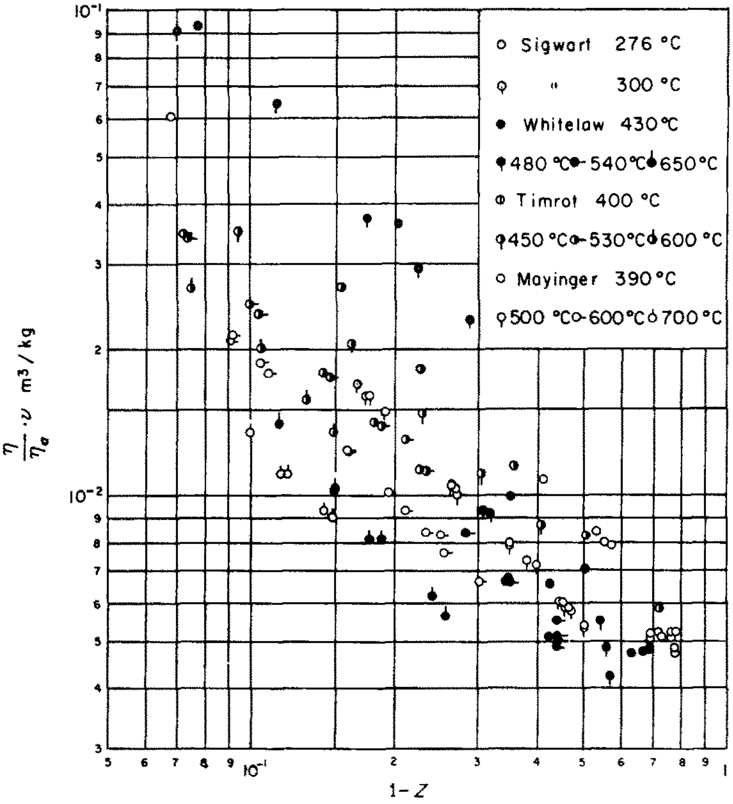

FIG. 3. Relation between viscosity at high pressures and compressibility factor.

equation of state of steam, the compressibility factor Z is given as follows,

$$Z = 1 + \frac{B}{v} + \frac{C}{v^2} + \dots \tag{6}$$

where, B in m³/kg, C in m⁶/kg² are the second and the third virial coefficients respectively and can be calculated by the following [8]

$$B = 1{\cdot}4122 \times 10^{-3} - 2{\cdot}9658\tau + 2{\cdot}0443 \times 10^3\,\tau^2 - 1{\cdot}7632 \times 10^6\,\tau^3 \tag{7}$$

$$C = 1{\cdot}156 \times 10^{-6} + 6{\cdot}722\tau^2 - 1{\cdot}161 \times 10^4\,\tau^3 + 5{\cdot}967 \times 10^6\,\tau^4 \tag{8}$$

where τ is the reciprocal absolute temperature, $1/T$.

Therefore, the compressibility factor at atmospheric pressure (indicated by subscripts a) approximately yields to the following formula

$$Z_a = 1 + \frac{B}{v_a}. \tag{9}$$

Then, the reduced viscosity η^* (namely the value η divided by the value η_a at atmospheric pressure) may be written as a function of a dimensionless quantity Y defined by

$$Y = [-(B/v)]/(1 - Z). \tag{10}$$

Fig. 4 shows the relation between the reduced viscosity and the dimensionless quantity Y. From this figure, it is considered that the reduced viscosity is equal to the quantity Y in the region of comparatively small values of quantity Y

$$\eta^* = Y. \tag{11}$$

But it seems that in the range of large values of quantity Y, namely in the range of low

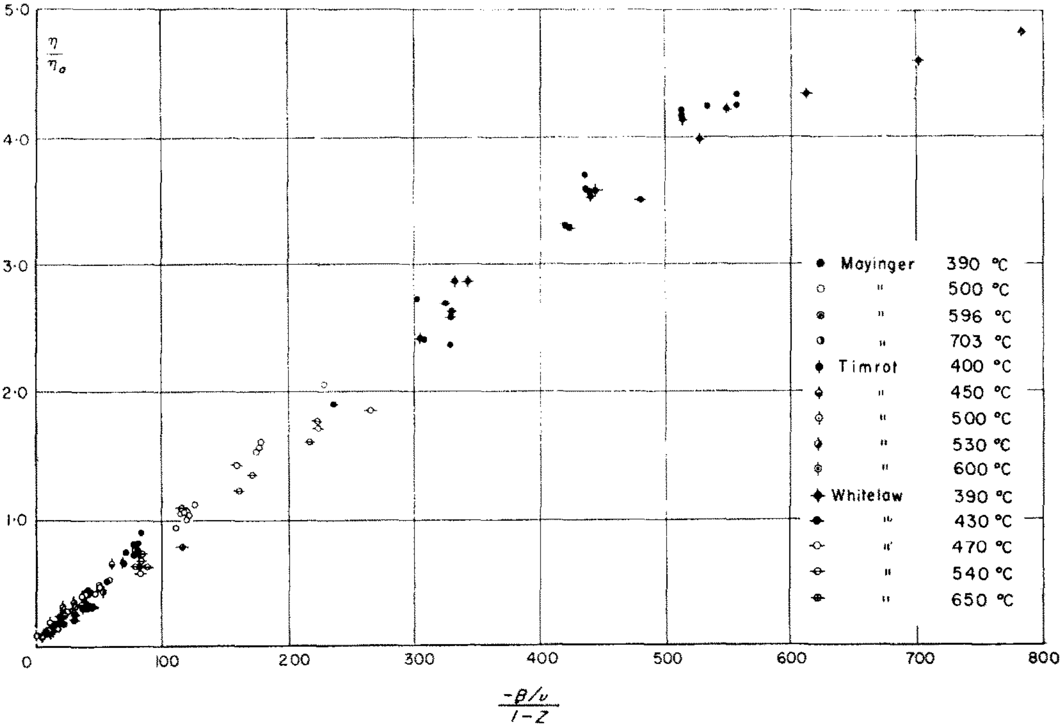

FIG. 4. Relation between η/η_a and $(-B/v)/(1-Z)$.

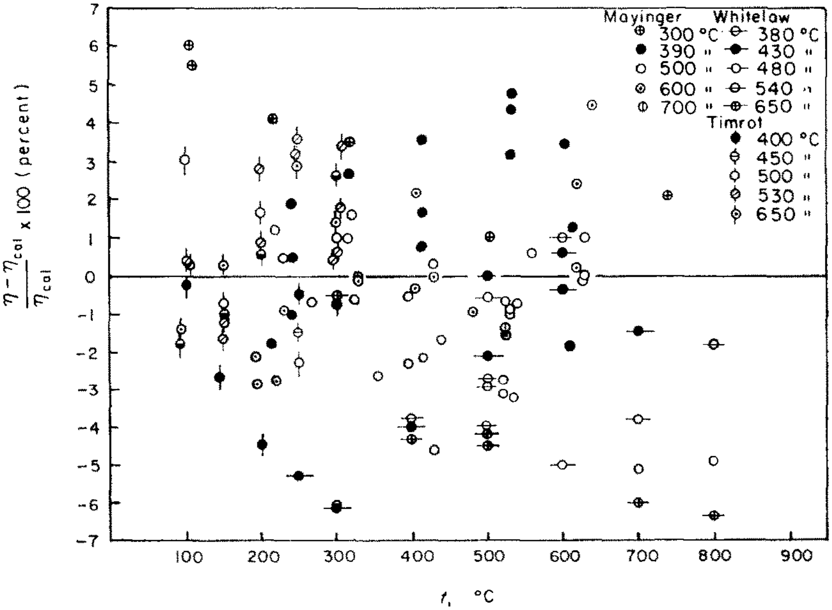

FIG. 5. Comparison of results at high pressures.

temperature and high pressure near the saturation line and the critical point, some correction terms ought to be added to the relation mentioned above.

Consequently, the following correlation between η^* and Y is obtained.

$$\eta^* = Y - F(T^*)\,(1 - Y)^2 \qquad (12)$$

where,

$$F(T^*) = 8{\cdot}474 \times 10^{-2}\,T^{*2{\cdot}9}. \qquad (13)$$

Then,

$$\frac{\eta}{\eta_a} = \frac{-(B/v)}{1-Z}$$
$$- 8{\cdot}474 \times 10^{-2}\left(\frac{T}{T_c}\right)^{2\cdot 9}\left[1 - \frac{-(B/v)}{1-Z}\right]^2. \quad (14)$$

This relation means that the viscosity of steam is connected closely to the property of steam as an imperfect gas. And it may be deduced that there exists a comparatively simple theoretical relationship between the viscosity and the other quantity of state also for polar gases.

Fig. 5 shows the comparison of the calculated values η_a by this method with the experimental results by Timrot, Whitelaw and Mayinger. The deviations between them are within about 5 per cent with the exception of several points by Whitelaw, and the Whitelaw's values have a somewhat lower tendency than the values by Timrot and Mayinger. The coincidence with the experimental values and the calculated values is reasonably good.

Although the viscosity does not depend on the density only, for steam also for water as mentioned above, it may be a noticeable method to

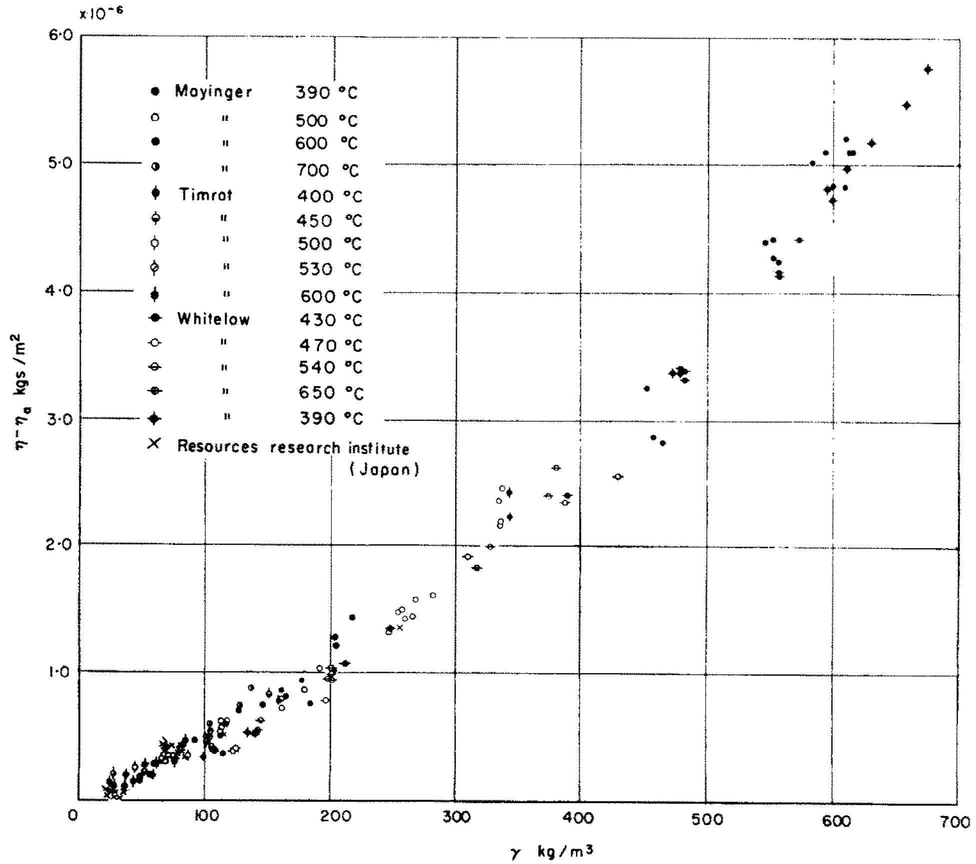

FIG. 6. Relation between ($\eta - \eta_a$) and γ.

consider the relation between the excess viscosity $(\eta - \eta_a)$ from the viscosity at atmospheric pressure at the respective temperature and the specific weight γ. In Fig. 6 is presented the relation between $(\eta - \eta_a)$ and γ. It follows from this figure that the excess viscosity is approximately the function of the specific weight only, and is expressed as follows,

$$\eta - \eta_a = a\left(\frac{\gamma}{100}\right) + b\left(\frac{\gamma}{100}\right)^2 + c\left(\frac{\gamma}{100}\right)^3. \quad (15)$$

The coefficients a, b and c were determined as follows so as to satisfy reasonably the values carried out by Timrot, Whitelaw and Mayinger

$$a = 5{\cdot}18 \times 10^{-7}$$
$$b = -2{\cdot}78 \times 10^{-9}$$
$$c = 9{\cdot}44 \times 10^{-9}.$$

Then,

$$\eta - \eta_a = 5{\cdot}18 \times 10^{-7}\left(\frac{\gamma}{100}\right) - 2{\cdot}78 \times 10^{-9}\left(\frac{\gamma}{100}\right)^2 + 9{\cdot}44 \times 10^{-9}\left(\frac{\gamma}{100}\right)^3. \quad (16)$$

But, as shown in Fig. 6, Mayinger's results along the four isotherms deviate from this curve systematically.

Fig. 7 shows comparison of the values calculated by equation (14) with the ones calculated by equation (16). Good agreement is found between the calculated values by (14) and the measured values by Mayinger particularly at high temperatures and high pressures.

The experimental investigation of the viscosity of steam in Japan was started in 1961 with a capillary viscometer by Tanaka, Sasaki *et al.* at the Resources Research Institute, and recently the results [9] up to 800°C and 1000 kg/cm² have been obtained along the following five approximate isobarics: 400, 500, 600, 800 and 1000 kg/cm².

The comparison of the calculated values given by equation (14) with the data at the Resources Research Institute is presented in Fig. 8. Fig. 9 also shows the comparison of the calculated given values by equation (16) with the same data. These values agree comparatively well with the values given by equation (14) with the exception that the values along the isobarics of 800 and 1000 kg/cm² have a different tendency and preferably agree very well with the values calculated by equation (16) as shown in Fig. 9.

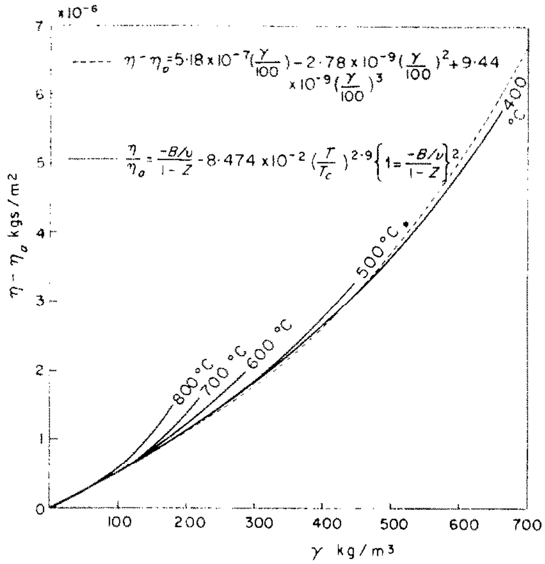

FIG. 7. Relation between equations (14) and (16).

It is questionable which is the most acceptable formula to express the behaviour of the viscosity of steam at high parameters in (14) or (16). And it must be decided by the more reliable values measured by the different methods with the more precise accuracy in the wide temperature and pressure range, and by the rigorous kinetic theory taking into consideration the polarizability for a dense gas composed of polar molecules.

But the difference between the values by equation (14) and by (16) is within about 4 per cent in the range of this comparison up to 800 kg/cm² and 800°C, and it makes no great difference practically either way.

The authors would rather prefer equation (14) as a viscosity formulation of steam.

The values of steam viscosity calculated by (14) are shown in Fig. 10 up to 800 kg/cm² and 800°C. In this computation, η_a, B, v and Z are calculated by using (4), (7) and Vukalovitch's

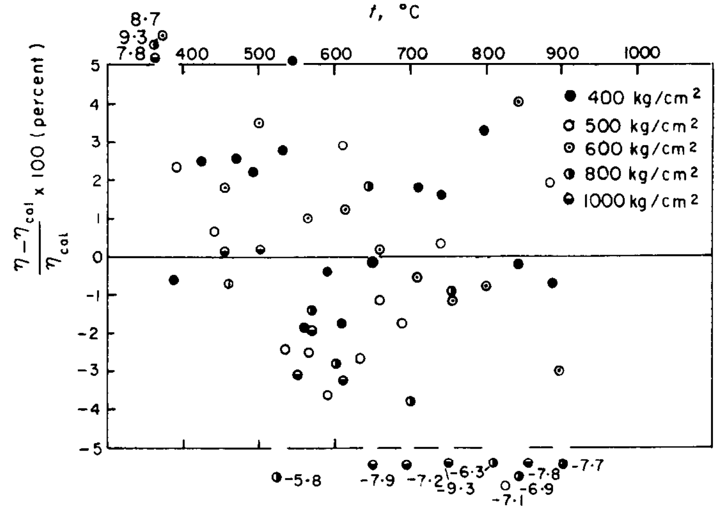

FIG. 8. Comparison of results [Resources Research Institute] with calculated values by equation (14).

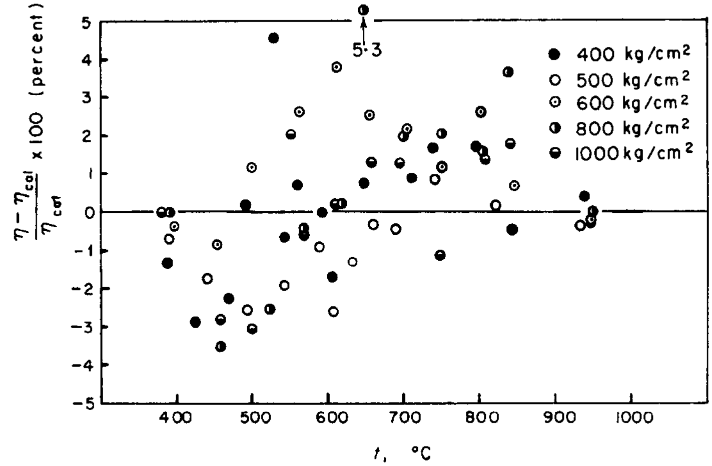

FIG. 9. Comparison of results [Resources Research Institute] with calculated values by equation (16).

steam table [10]. As shown in Fig. 10, the viscosity of steam increases in accordance with pressure rise under constant temperature but in the lower range of pressure than about 100 kg/cm², the effect of pressure is comparatively small.

On the other hand, as regards the effect of temperature upon the viscosity of steam, the viscosity increases uniformly with temperature rise in low pressure range, but in high pressure range especially over the critical pressure, the viscosity of steam has a minimum value along each isobarics. And the viscosity in the region of pressurized water decreases in accordance with temperature rise.

The steam viscosity at the critical point is given by using $Z_c = 0{\cdot}2415$, $v_c = 0{\cdot}0326$ m³/kg in equation (14) as follows,

$$\eta_c = 4{\cdot}623 \times 10^{-6}\ \text{kg s/m}^2.$$

And also the viscosity of saturated steam and water and the kinematic viscosity can be calculated by using equation (14).

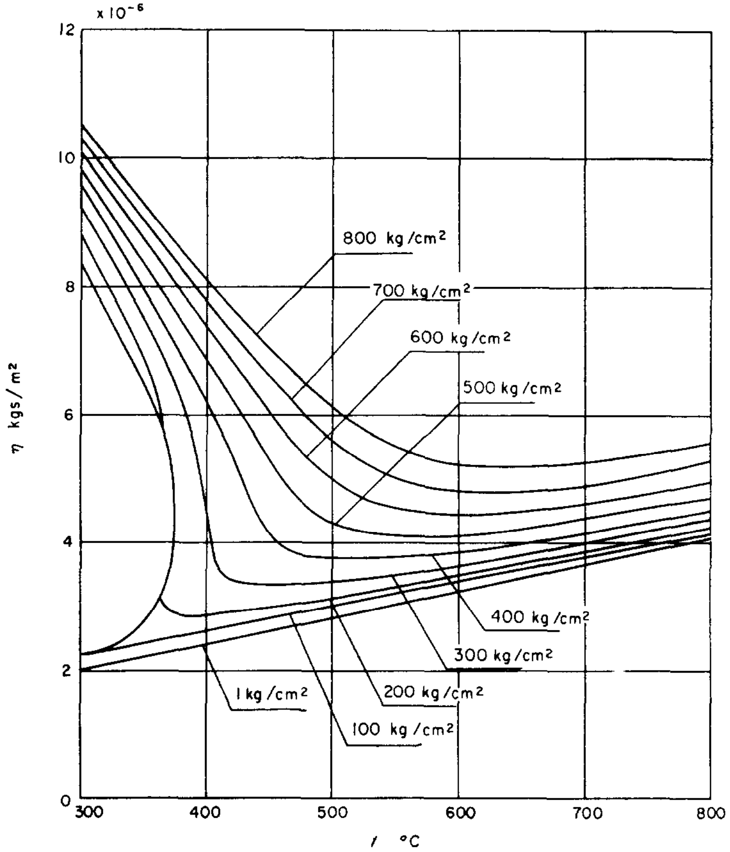

FIG. 10. Viscosity diagram of steam at high pressures.

4. CONCLUSION

1. The dimensionless relation between the viscosity of steam and temperature at atmospheric pressure was determined on the basis of the various values reported by several investigators.

2. Based on the various experimental results, the relation between the reduced viscosity of steam and the dimensionless quantity Y composed of the compressibility factor and the second virial coefficient was established, and compared with the relation between the excess viscosity and specific weight. Moreover, the dependence of the viscosity of steam upon pressure and temperature was made clear up to 800°C and 800 kg/cm^2.

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Résumé—La viscosité d'un gaz réel dépend non seulement de la température mais aussi de la pression, c'est-à-dire qu'elle peut être considérée comme une sorte de propriété d'état liée aux autres par une relation. Pour établir clairement la relation entre viscosité, température et pression, considérée de cette façon, cet article traite numériquement les résultats expérimentaux de nombreux chercheurs. Une relation nouvelle est établie entre la viscosité, le second coefficient du viriel et le facteur de compressibilité.

Zusammenfassung—Die Zähigkeit eines realen Gases hängt nicht nur von der Temperatur, sondern auch vom Druck ab; sie könnte eine Art Zustandsgrösse sein und über eine Beziehung mit anderen Zustandsgrössen verbunden sein. Um die Abhängigkeit der Dampfviskosität von Druck und Temperatur in dieser Hinsicht zu untersuchen, wurden die Versuchsergebnisse verschiedener Forscher in vorliegender Arbeit einer numerischen Behandlung unterzogen. Eine neue charakteristische Dampfbeziehung zwischen Viskosität, dem zweiten Virialkoeffizienten und dem Kompressibilitätsfaktor wurde aufgestellt.

Аннотация—Вязкость реального газа зависит не только от температуры, но также и от давления, т.е. она может представлять собой вид параметра состояния, и находится в какой-то связи с другими параметрами состояния. Чтобы выяснить зависимость вязкости пара от давления и температуры с этой точки зрения, в статье приводится численная обработка экспериментальных данных, полученных различными исследователями. В результате, для пара установлена новая характеристическая связь между вязкостью, вторым вириальным коэффициентом и коэффициентом сжимаемости.